AOS Application Note: Specifying Wavefront or Surface Error in Aspheres



#### INTRODUCTION

The most common tolerances for specifying the optical quality of aspheric mirrors such as off axis parabolas are surface accuracy and reflected wavefront error. Surface error is the deviation of the surface from its perfect form. Wavefront error is the deviation of the resulting reflected or transmitted wavefront from its perfect shape.

At first glance, the decision to specify optics based on its wavefront quality or surface form may seem purely one of preference. One might be tempted to assume they are merely related to each other by a scale factor. However, this is not correct. One can convert between the two – but not by a simple scale factor like in a flat or spherical optic. It turns out that the decision can be quite important when one considers how precision optics are measured. It may surprise some people to know that interferometers measuring aspheres in typical test configurations *do not* measure surface form error at all – only wavefront error and there is no single scale factor that can extract surface error for an off-axis parabola. The consequence of incorrect scaling is that those specifying optical quality based on surface accuracy may not be getting the quality they believe.

#### SURFACE ACCURACY VS. WAVEFRONT ERROR

Although Surface Form Error or "Surface Accuracy" is often specified to describe the quality of common aspheric mirrors like off-axis parabolas, surface accuracy cannot be directly measured interferometrically in a typical null configuration – only the reflected wavefront error (RWE) can be measured. In a flat or a spherical surface, conversion between the two is a straightforward scale factor based on the angle of incidence and a factor of 2 (for double pass tests)<sup>i</sup>. However in an off-axis parabola, the null configuration test is based on alignment with respect to the off-axis angle – not the angle of incidence (AOI). These are not the same. Furthermore, there is no constant AOI in an off-axis parabola. Rather the AOI varies as a function of zonal radius. Therefore, unlike a spherical or flat surface, the conversion factor relating the measured wavefront data to surface error is a function – not a scale factor.



figure 1: Relationship between wavefront error and surface error in a sphere

As of the writing of this paper, the most common wavefront analysis software packages make no accommodation for this conversion without writing a user-defined transformation. Therefore,

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many make the mistake of incorrectly scaling the data using the off-axis angle as if the optic surface where spherical. If the departure from a sphere in the measured asphere is small, then this error in the calculation is similarly small and it may be OK to neglect. However as the aspheric departure becomes large, more error in the reported values exist. We'll examine the case for a sphere and a parabolic mirror to illustrate the difference.

Figure 1 illustrates the case for a spherical mirror. To test a spherical surface with an interferometer, one aligns the sphere such that its radius of curvature and the focus of the reference element (transmission sphere) overlap. In this configuration, all rays from the interferometer and incident upon the surface are auto-reflected normal to the tangent of the surface. The reflected wavefront error resulting from surface errors is exactly two times the depth of the surface flaw because the reference wavefront traverses the flaw going in and coming back. This is the typical double pass test, and the scale factor relating wavefront error to surface errors is "2".

Figure 2 illustrates this case for a parabolic mirror. In this case, the parabola is aligned to the focus of the reference sphere at its focal length (not radius of curvature). The reflected wavefront is thus not reflected normal to the surface tangent as in the spherical mirror. By definition of a parabola, the reflected rays are now collimated and must be auto-reflected from a reference flat mirror (RF). The factor relating wavefront error to surface error =  $1/2(\cos\theta)$ . The question is what is  $\theta$ ? It is not constant. Based on the geometry of off-axis parabolas<sup>ii</sup>, we've derived the incident angle as a function zonal radius (r) to be:

$$\theta_i = 90 - tan^{-1}(\frac{2f_p}{r})$$

where  $\theta_i$  = the angle of incidence

 $f_p$  = parent focal length

 $f_s$  = sectional focal length

r = zonal radius (distance from parent axis)



figure 2: Relationship between wavefront error and surface error in an off-axis parabola

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## **APPLICATION EXAMPLE**

The next question might be: Is it significant?

Consider the case for a 300-mm diameter f/2, 90-degree off-axis parabola with  $\lambda/4$  (158 nm) of reflected wavefront error. The spreadsheet shown in figure 3 was generated to calculate the range of incident angles for this parabola when tested in the way the parabola is used.

		Angle of		
	Zonal	Incidence		
	Radius (mm)	(degrees)	Constants	(mm)
Lower Bound	450.00	36.87	Sectional Focal Length	600
	480.00	38.66	Off Axis Distance	600
	510.00	40.36	Clear Aperture	300
	540.00	41.99		
	570.00	43.53	Calculated	
	600.00	45.00	Off Axis Angle	90
	630.00	46.40		
	660.00	47.73		
	690.00	48.99		
	720.00	50.19		
Upper Bound	750.00	51.34		



The range of calculated incident angles are 36.9 degrees to 51.3 degrees.

When surface error is calculated incorrectly with a constant angle of incidence of 45 degrees, the result is 112 nm. However, when calculated with the correct worst case incident angle the same reflected wavefront would predict a surface form error of 127 nm, a difference of 13%. For slower and smaller OAPs some might not think the difference is important. But why be wrong when its easy to be correct?

There are at least four reasons why surface form error specifications still persist in OAP designs and catalogs.

- 1. Surface form error can be measured using surface profilometry. This is one way manufacturers have provided quality data in the past and one of the most universally available methods. The only problem with this is that most surface profilometers do not have comparable precision to interferometry. So this method is not recommended for high precision OAPs.
- 2. Because Reflected Wavefront Error tests are generally double or 4-pass tests, surface form errors are smaller numbers than reflected wavefront error values. Therefore, there is a perceived (but incorrect) implication of superior quality.
- 3. In order to optimize wavefront quality, manufacturers must correct for surface errors during finishing. Therefore, it is the language most common for optical polishing. However, even still, manufacturers using interferometry can still only measure wavefronts.
- 4. Tolerancing and measurement of aspheres are not as widely understood as are flats and spheres today and honest misunderstanding still persists.

The best reason for dismissing surface accuracy tolerances however, is because, reflected wavefront error is an optical metric and is more directly related to image quality and the functional performance of mirrors. In other words, it is the wavefront error, which determines

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image quality, and it is the wavefront error that manufacturers measure, so there is no good reason anymore to adopt specifications for surface accuracy in off-axis parabolas.

## **CONCLUSIONS & RECOMMENDATIONS**

- 1. Surface Error and Wavefront Error are not so easily scaled for aspheres as they are for flats and spherical optics because they are tested in fundamentally different ways.
- 2. Don't specify the optical quality of off-axis parabolas using surface error tolerances, use reflected wavefront error instead.
- 3. Question any interferometry that claims to represent surface error measurement

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# AUTHOR



Flemming Tinker, President, Aperture Optical Sciences Inc.

Flemming Tinker has been working in the field of optics engineering and manufacturing for over 30 years and holds degrees in Optics from the University of Rochester, and Manufacturing Engineering from Boston University. Mr. Tinker is a specialist in optics manufacturing & management involving innovative technologies.

<sup>&</sup>lt;sup>i</sup> D. Malacara, Optical Shop Testing, Third Edition, Wiley-Interscience, John Wiley & Sons 2007 p9.

<sup>&</sup>lt;sup>ii</sup> http://www.apertureos.com/products/aspheres/off-axis-parabolas/